Brain and mathematical cognition

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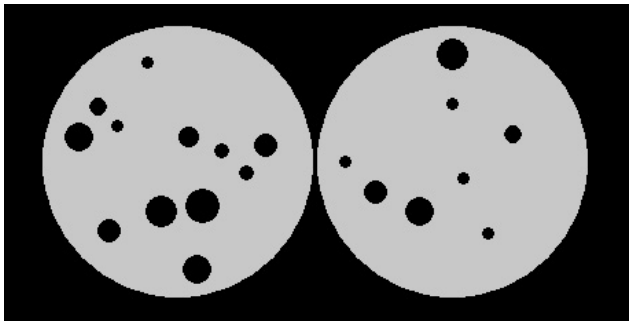
Outline

- 1. Quantity processing in brain
- 2. Calculation in brain
- 3. Mathematical reasoning in brain
- 4. Training and mathematical brain

1. Quantity processing in brain

- Non-symbolic quantity (Numerosity/magnitude)
- Symbolic quantity

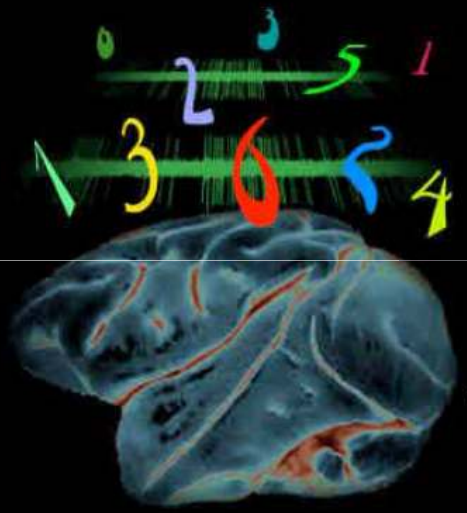


(www.dweipsy.com/lattice/)

Neuroscience of Number Processing

Single-Cell Neurophysiology in the Monkey

Andreas Nieder



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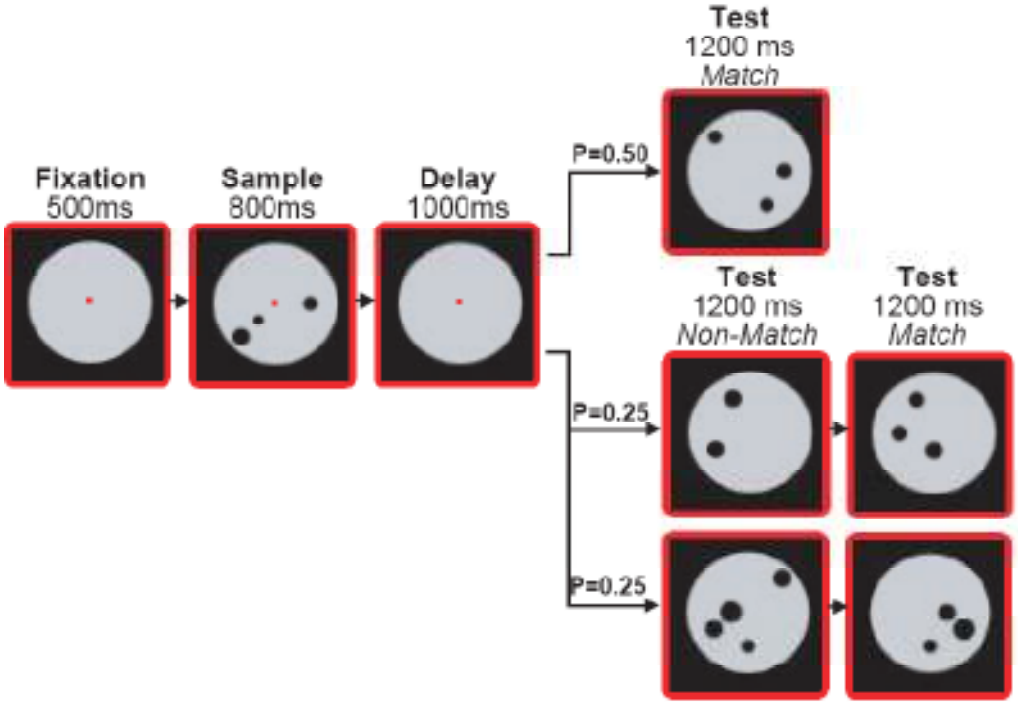
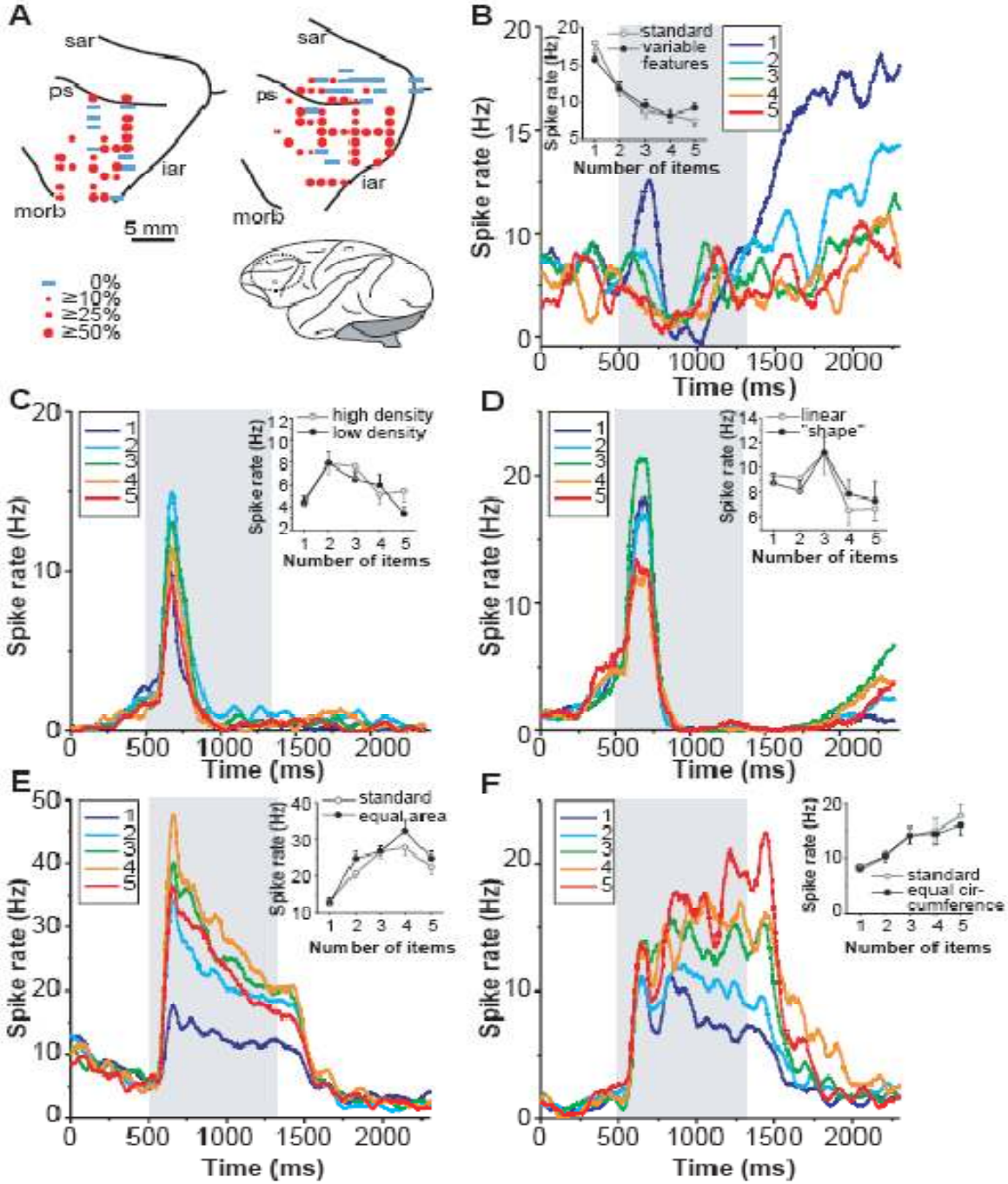
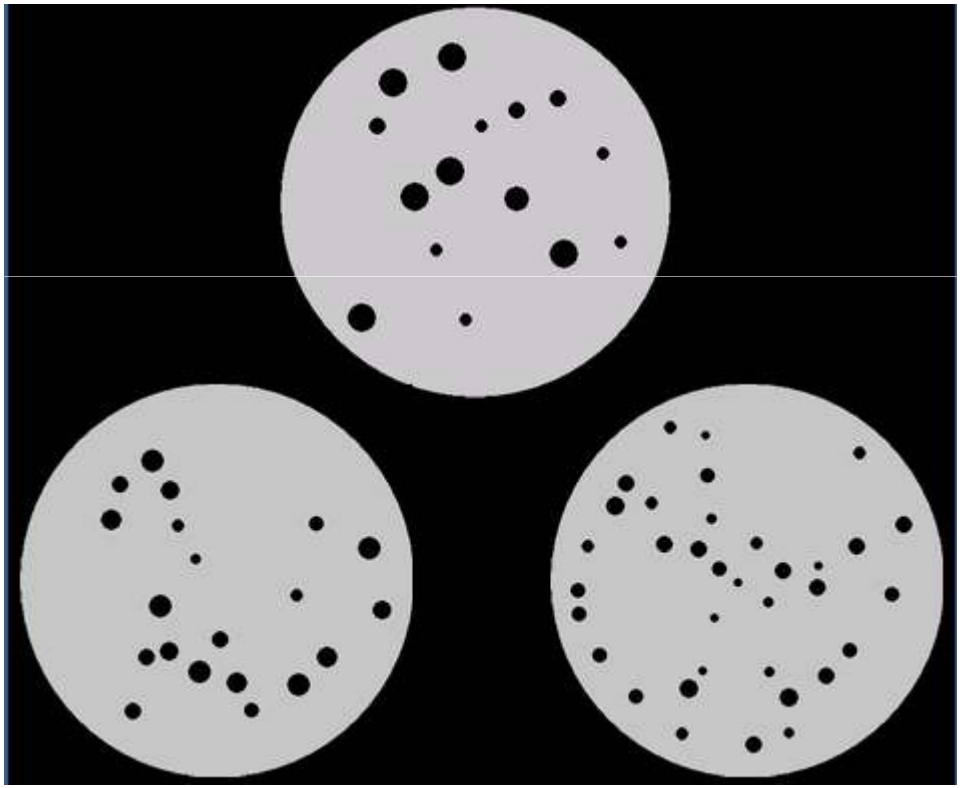
Representation of the quantity of visual items in the primate prefrontal cortex

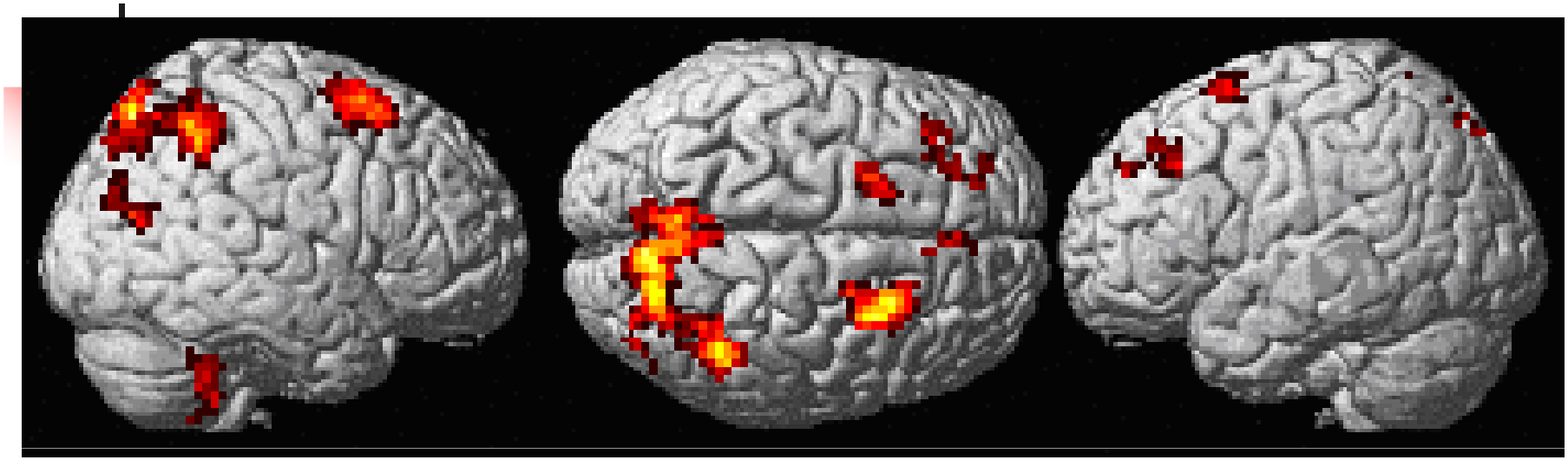


(Nieder et al., 2002, *Science*)


A
sar
ps
iar
morb
5 mm
0%
≥10%
≥25%
≥50%
B
standard
variable features
Number of items
Spike rate (Hz)
Time (ms)
C
high density
low density
D
linear
"shape"
E
standard
equal area
F
standard
equal circumference

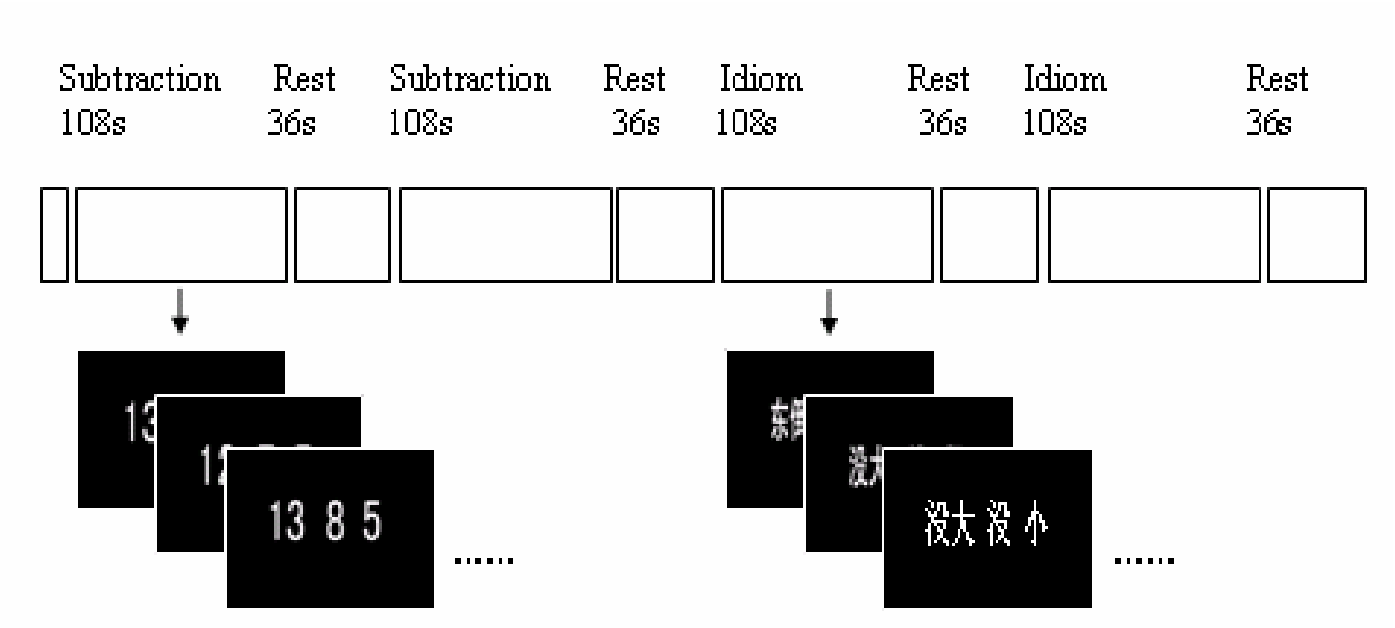
Numerosity comparison in human brain



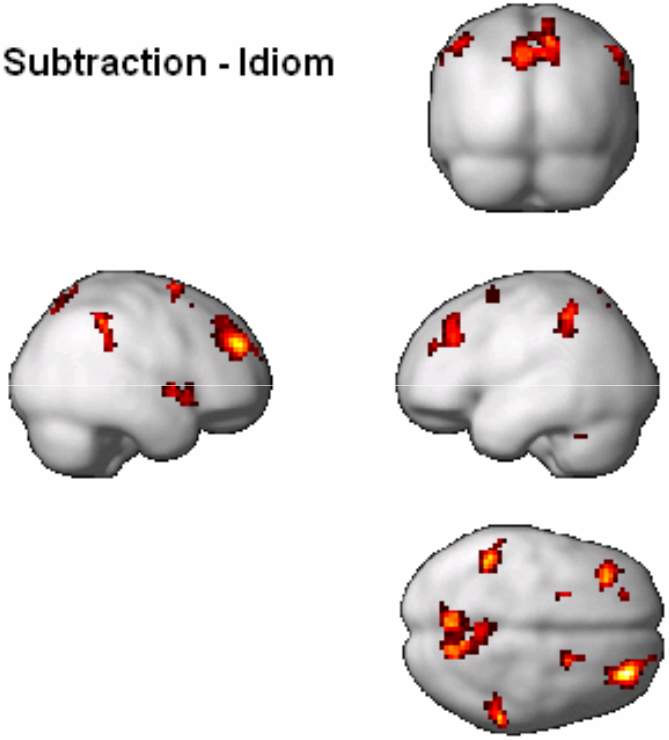


- 2. Calculation in human brain

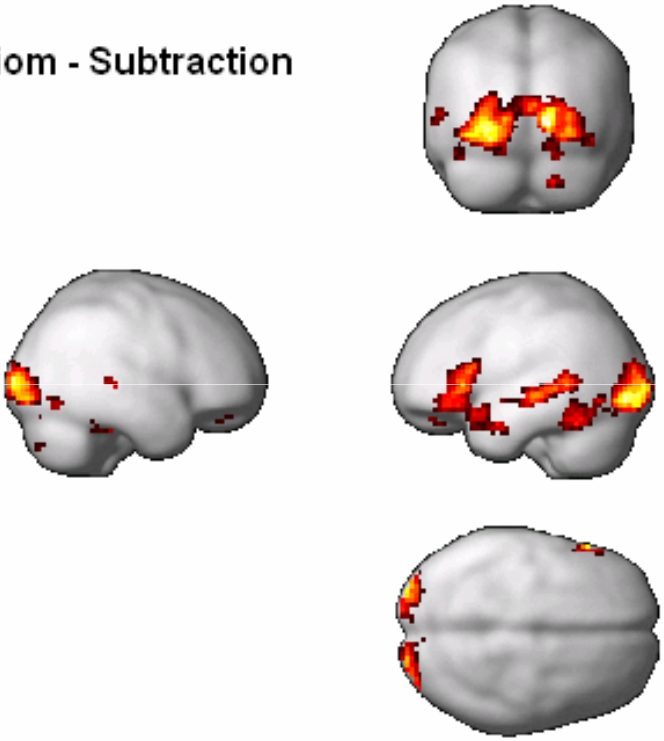
Subtraction and idiom processing



Subtraction - Idiom



Idiom - Subtraction



Agrammatic but numerate

- Three men with large left-hemisphere perisylvian lesions.
- Despite severe grammatical impairment and some difficulty in processing phonological and orthographic number words, all basic computational procedures were intact across patients.

(Varley et al., 2005, PNAS)

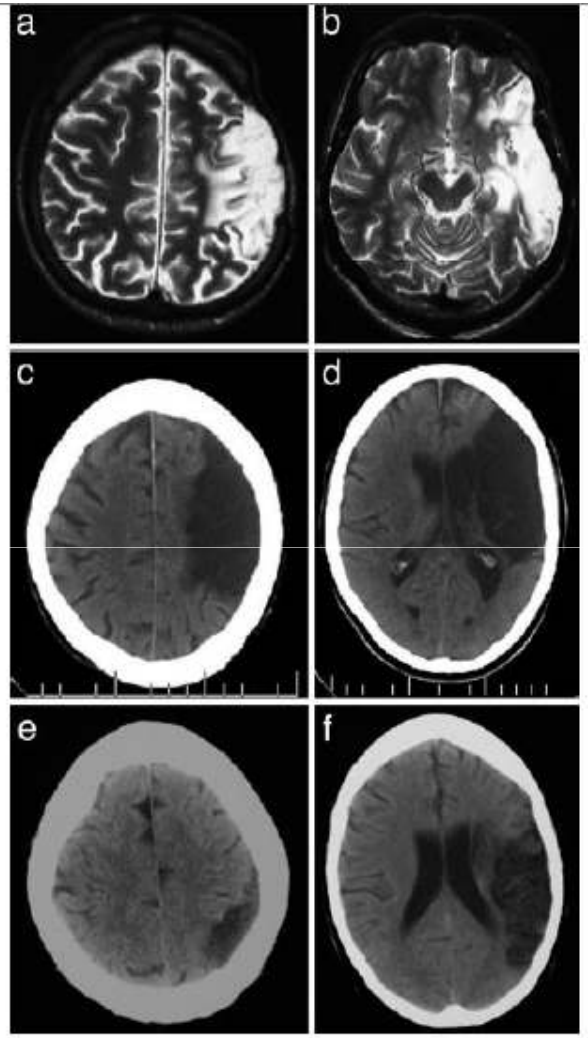
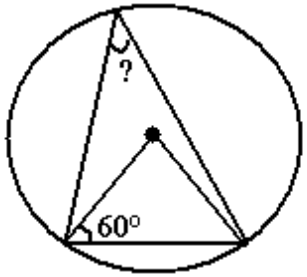
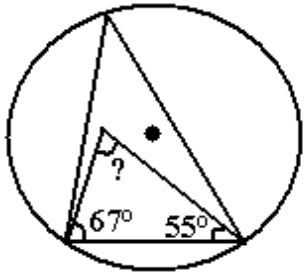


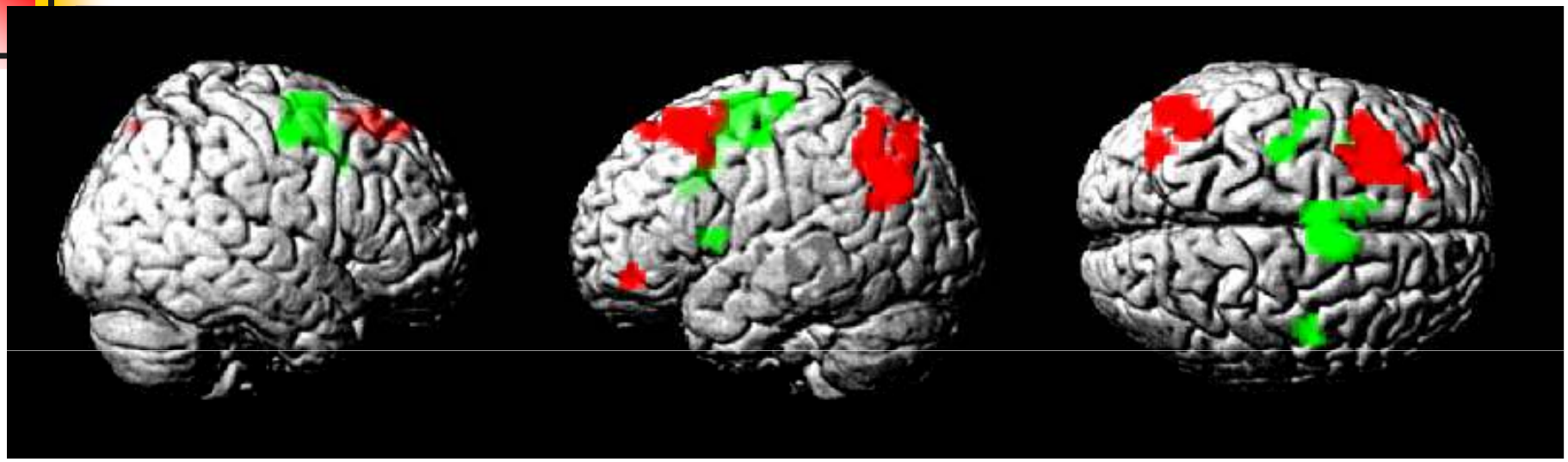
Fig. 1. Structural brain scans for patients S.A. (*a* and *b*), S.O. (*c* and *d*), and P.R. (*e* and *f*).

- They were unable to differentiate between the statements "Mary hit John" and "John hit Mary." However, these same patients successfully solved mathematical operations that were structurally dependent in this same general way, for instance, the difference between 52 - 11 and 11 - 52.
- The patients were unable to comprehend sentences with embedded clauses such as, "This is the dog that worried the cat that ate the rat that ate the malt that lay in the house that Jack built"; nonetheless, their performance was unimpaired in computing expressions with similar embedding, for example, answers to a sequence such as 90 - (3 +17) x 3.

- 3. Mathematical reasoning in brain

Mathematical problem solving and numerical calculation rely on differential brain structures (Lu et al., submitted)

	Mathematical problem solving	**Numerical calculation**
Number series	81:64:49:36	15:10:16:14
	32 30 25 24	55 65 56 54
Geometric problem		
	20° 25° 40° 30°	58° 47° 69° 49°
Word problem	Lucy has 90 marbles. Her brother has 60 marbles. How many marbles must she give to him to have an equal amount?	Lucy has 96 marbles. Her brother has 68 marbles. How many more marbles does she have than her brother?
	15 20 25 30	37 28 30 35



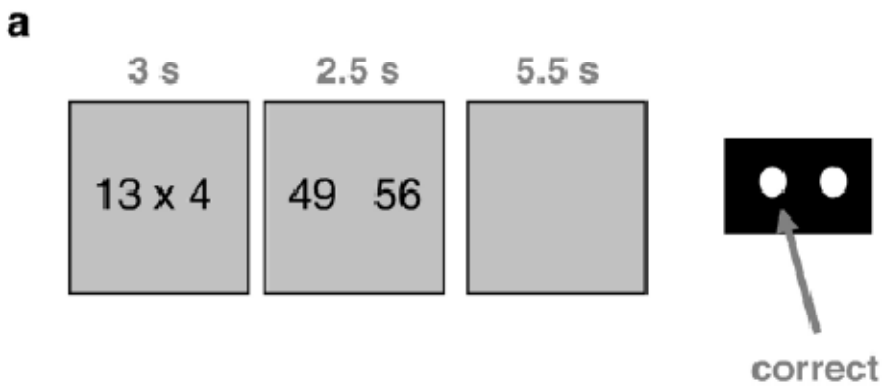
Mathematical problem solving vs. numerical calculation

- 4. Training and mathematical brain

Imaging practice effects in arithmetic

- Training consisted in a higher frequency of repetition for one set of complex multiplication problems (repeated) and a lower frequency for the other set (novel).
- Repeated and novel problems were presented randomly in an event-related design.

(Ischebeck et al., 2007, Neuroimage)



Choose the closest to solution

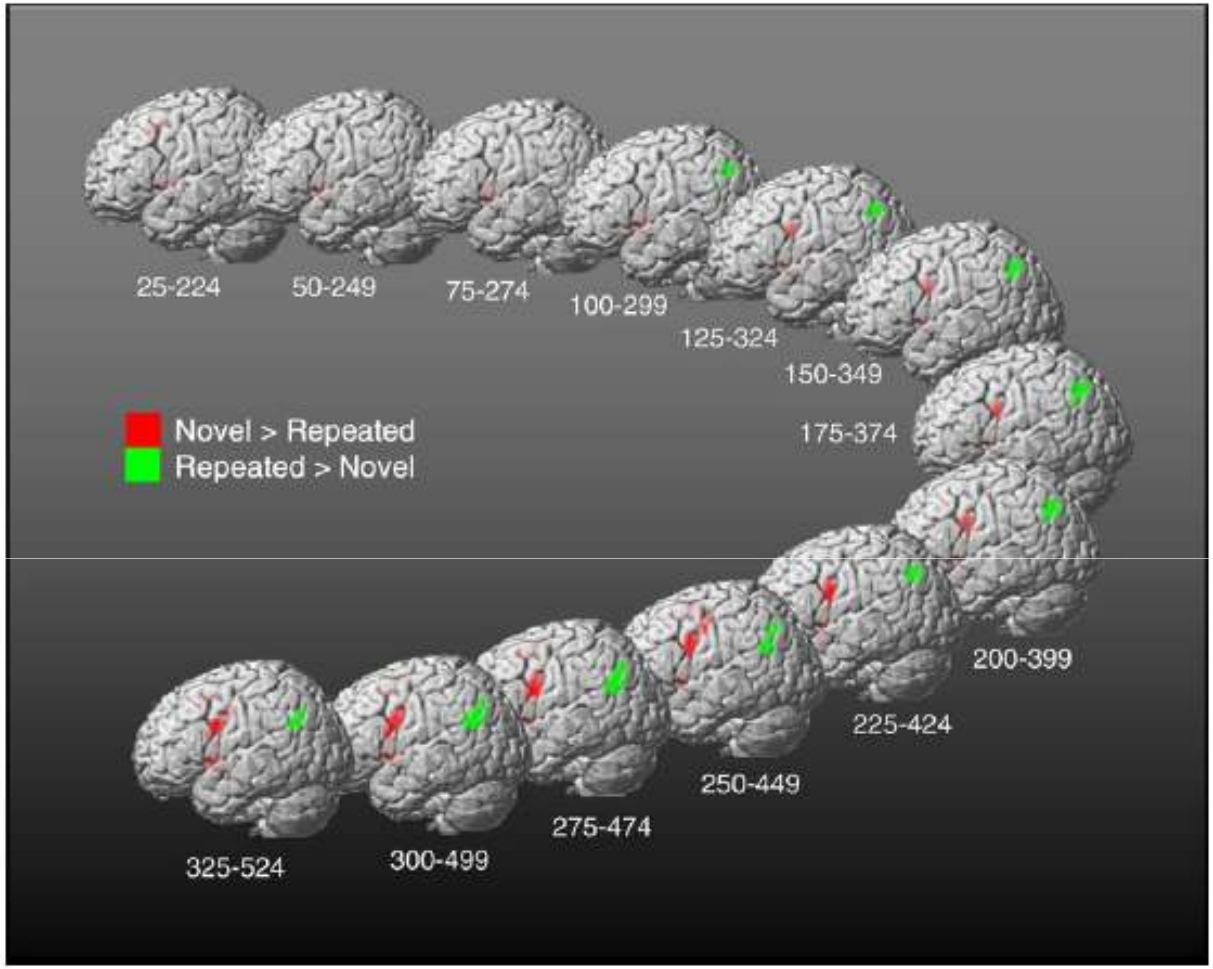
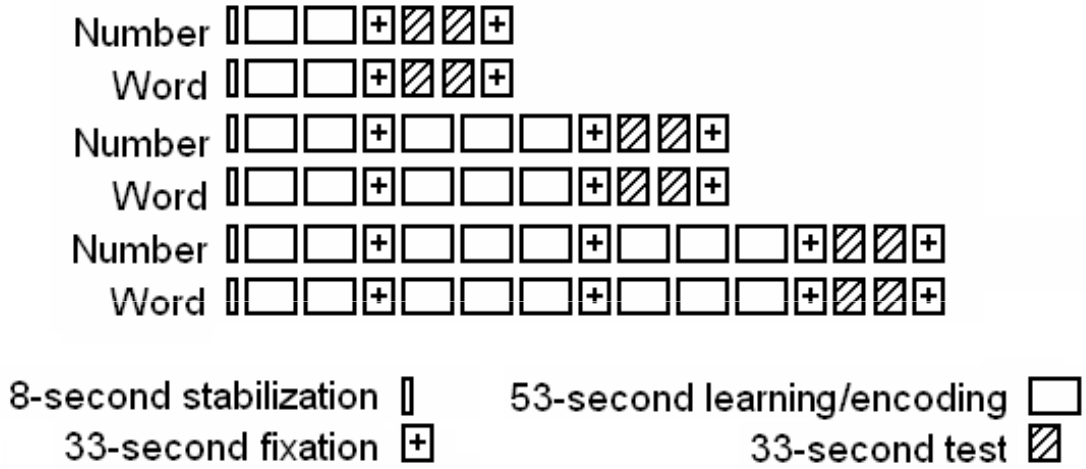


Fig. 2. Activation change due to practice over the course of the experiment. A moving window of 200 scans was analyzed. Repeated problems are compared to novel problems. Training effects (green: repeated > novel, red: novel > repeated) become significant beginning with the time window from 100 to 299 scans. Images are thresholed with an initial threshold of $p < 0.001$ uncorrected, showing only clusters with more than 30 voxels, corresponding to a $p < 0.05$ corrected on cluster level.

Neural correlates of effective number associate learning

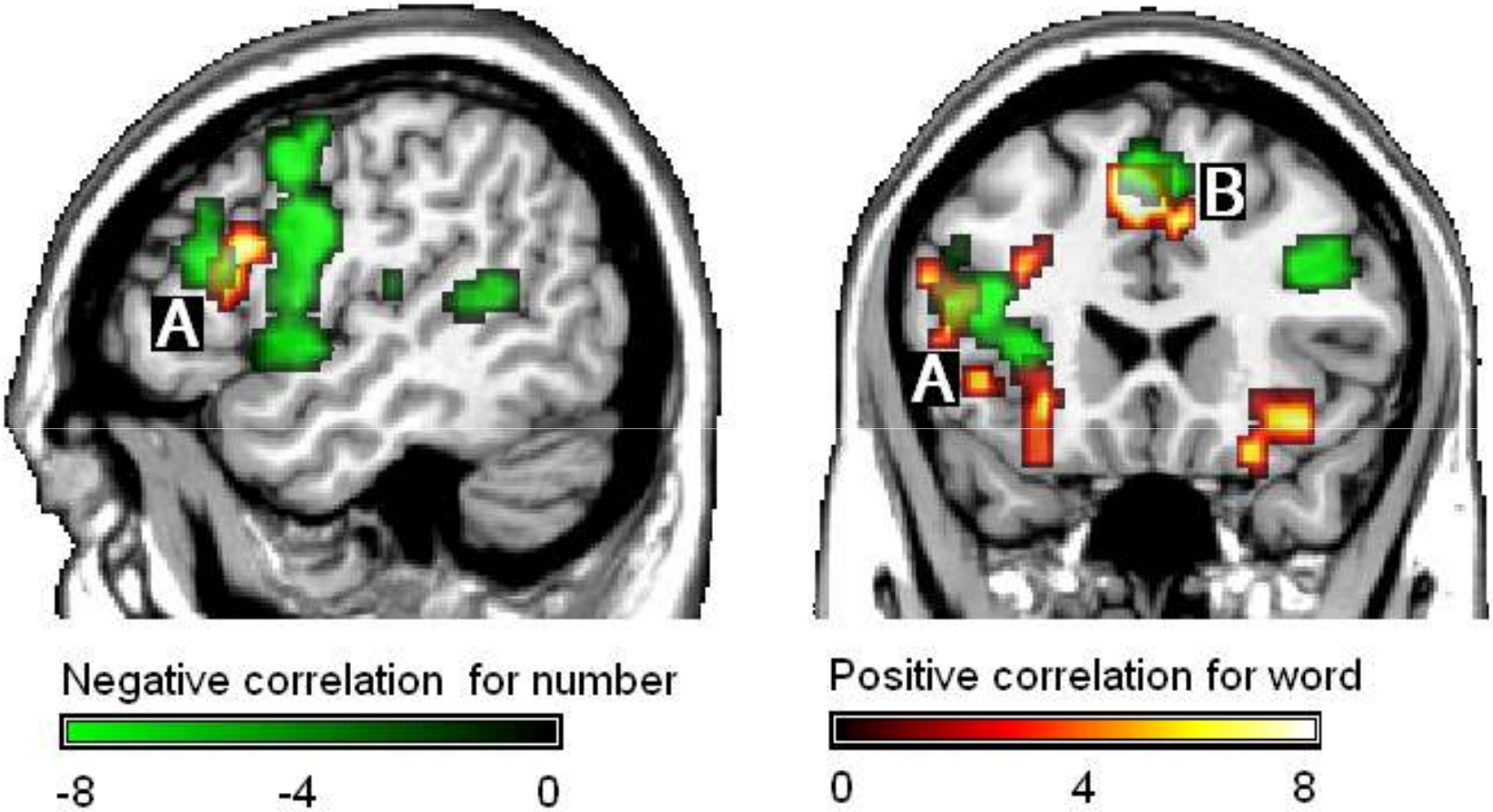


0 1 2 3 4 5 6 7 8 9

水 刀 马 灯 门 火 人 口 路 山

Number associate learning: 5 4 → 73

Word associate learning: 火 花 → 刀口


A
B
A
Negative correlation for number
-8
-4
0
Positive correlation for word
0
4
8

Increased grey matter density in the Parietal cortex of mathematicians

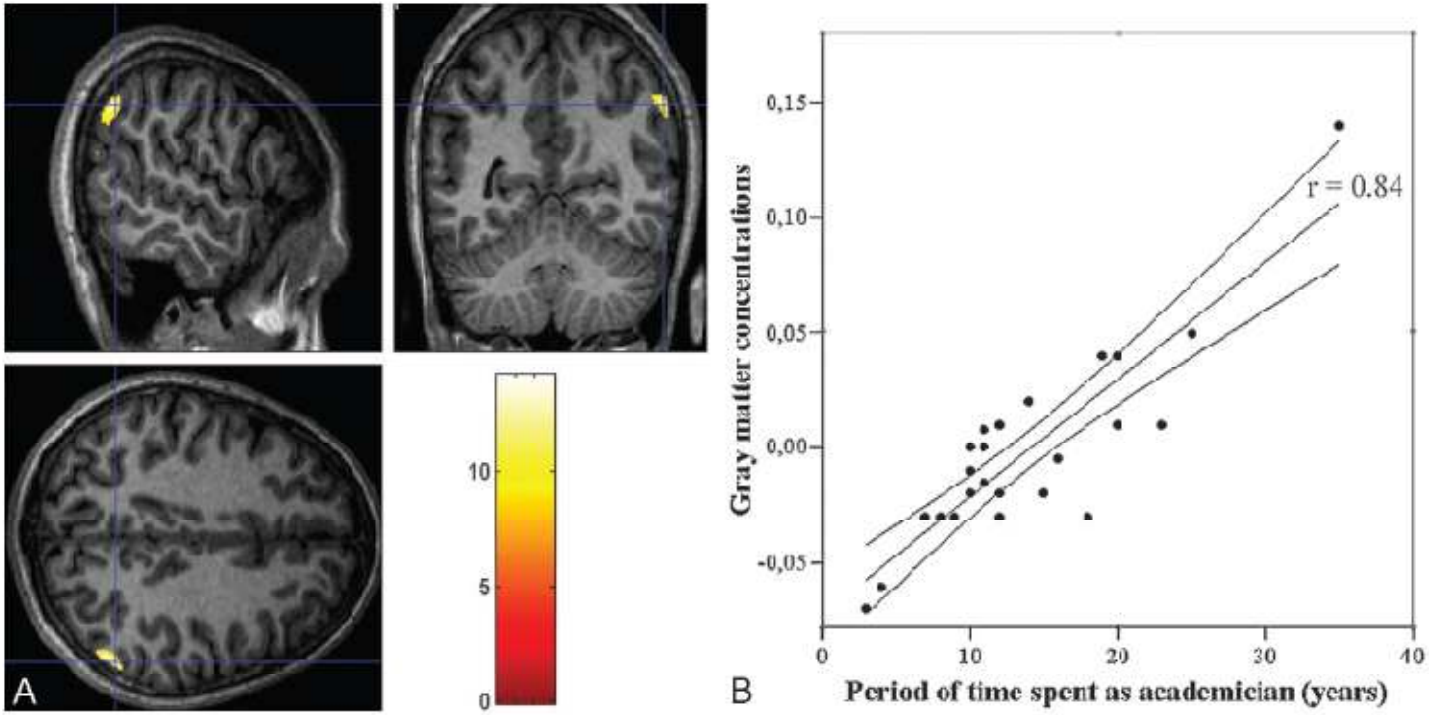


Fig 2. *A*, The result of the regression analysis testing the correlation between gray matter density of the mathematicians and period of time spent as an academician is overlayed on the normalized T1-weighted images. They show that gray matter density in the right inferior parietal region (Talairach coordinates: $x = 57$, $y = -60$, $z = 34$) of the mathematicians is strongly correlated with the duration of time spent as an academician ($z = 7.28$; $P < .05$, FWE corrected). *B*, The gray matter density values from the voxels showing the maximum correlation on the SPM regression analysis are extracted into SPSS. The scatter-plot graph shows the linear regression between the gray matter density and the duration of time spent as an academician ($r = 0.84$; $P < .01$). The *middle line* represents the linear regression, and the *curves* around it represent the 95% confidence intervals.

(Aydin et al., 2007, *Am J Neuroradiol*)

Dissociated brain organization in arithmetic processing

- Differential learning experience
  - Procedure strategies for addition facts (7+8=8+2+5=15)
  - Verbal strategy for multiplication facts (7x8=56, “七八五十六”)

- Differential neural basis for addition and multiplication

里耶秦简九九表

九九八十一	八九七十二	七九六十三	六九五十四	五九四十五
四九卅六	三九廿七	二九十八	八八六十四	七八五十六
六八四十八	五八四十	四八卅二	三八廿四	二八十六
七七四十九	六七四十二	五七卅五	四七廿八	三七廿一
二七十四	六六卅六	五六卅	四六廿四	三六十八
二六十二	五五廿五	四五廿	三五十五	二五而十
四四十六	三四十二	二四而八	三三而九	二三而六
二二而四	一一而二	二半而一		

凡千一百一十三字

Ancient multiplication table (from Qin Dynasty to Song Dynasty)

考古发现及传世典籍中的九九表

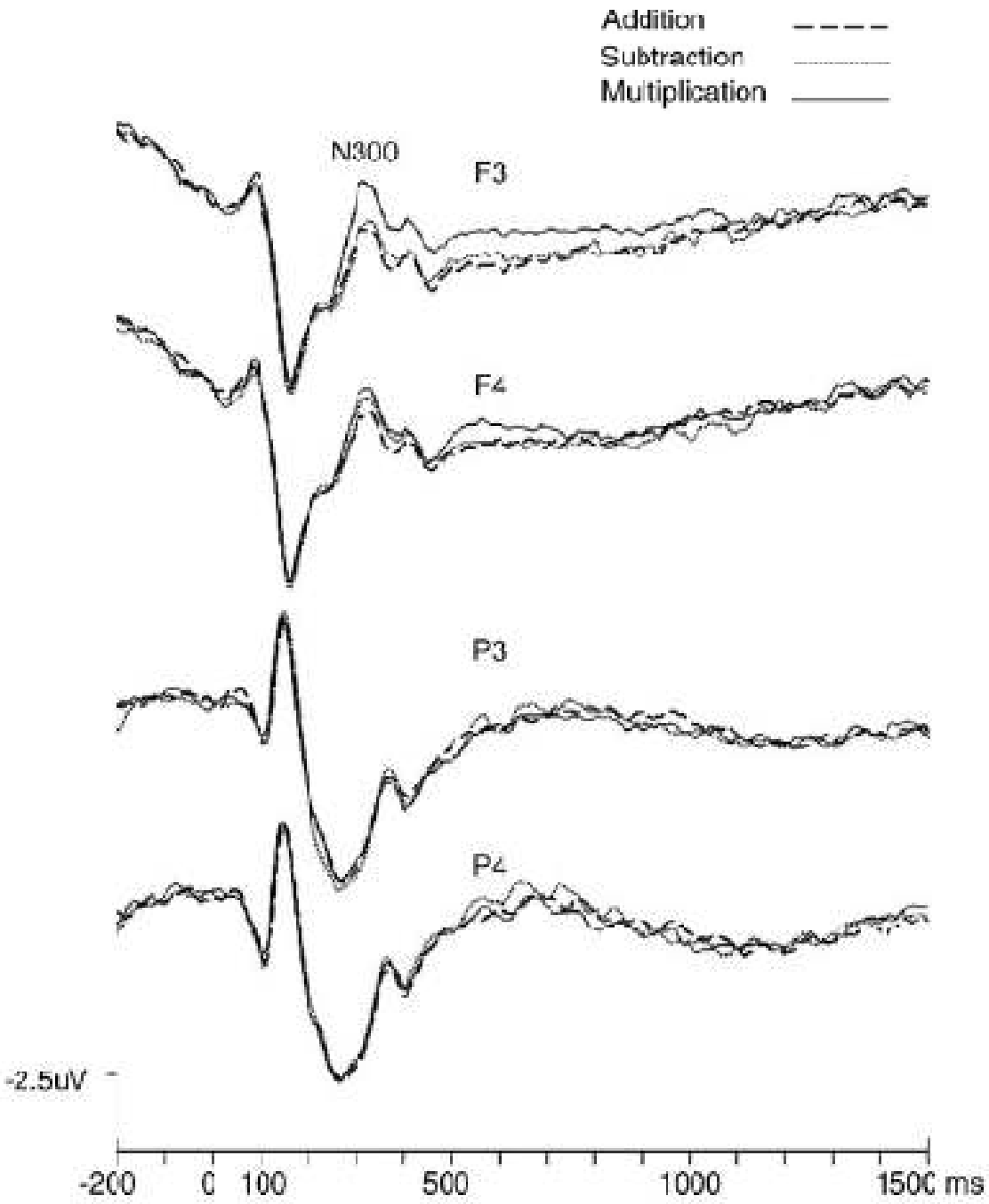
名　　称	时　代	材质	备　　注
里耶秦简九九表（全）	秦	木牍	2002年出土。始于"九九八十一"。参《考古》2003年第1期。
张家界汉简九九表(残)	汉	木简	2002年出土。始于"九九八十一"。参《中国历史文物》2003年第3期。
敦煌汉简九九表(残)	汉	木简	20世纪初发现。始于"九九八十一"。参王国维《流沙坠简》。
居延汉简九九表1(残)	汉	木简	1930年出土。始于"九九八十一"。参劳干《居延汉简考释》。
居延汉简九九表2(残)	汉	木简	1930年出土。始于"九九八十一"。参《居延汉简甲编》。
《孙子算经》九九表	汉 （据戴震）	纸张	传世典籍。始于"九九八十一"。参李俨《中国古代数学史料》。
敦煌卷子九九表1（残）	唐	纸张	20世纪初发现。始于"九九八十一"。参李俨《中国古代数学史料》所录敦煌《算经》。
敦煌卷子九九表2（全）	唐	纸张	20世纪初发现。始于"九九八十一"。参李俨《中国古代数学史料》所录敦煌《立成算经》。
楼兰文书九九表（残）	唐	纸张	20世纪初发现。始于"九九八十一"。参孔好古《斯文赫定所获汉文手写文书及零星文物》。
通行乘法口诀表	宋元以后	纸张	始于"一一如一"。参钱宝琮《中国数学史》。

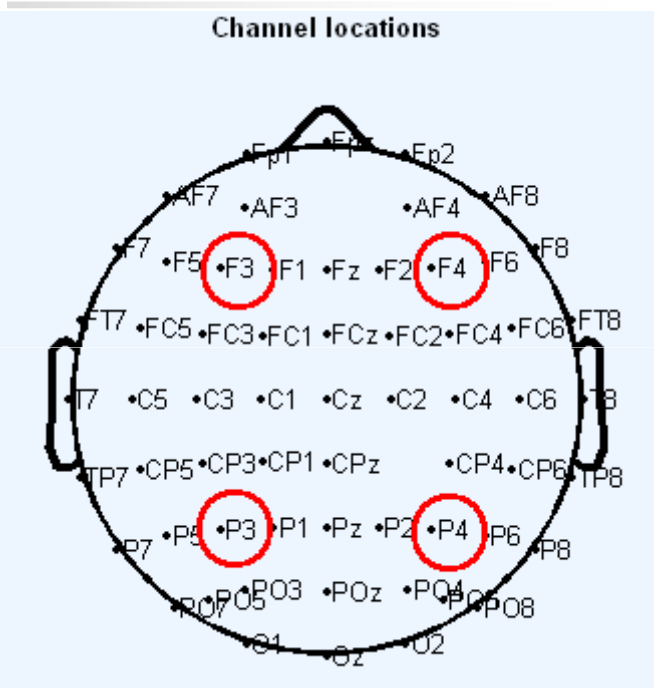
一一得一							
一二得二	二二得四						
一三得三	二三得六	三三得九					
一四得四	二四得八	三四十二	四四十六				
一五得五	二五一十	三五十五	四五二十	五五二十五			
一六得六	二六十二	三六十八	四六二十四	五六三十	六六三十六		
一七得七	二七十四	三七二十一	四七二十八	五七三十五	六七四十二	七七四十九	
一八得八	二八十六	三八二十四	四八三十二	五八四十	六八四十八	七八五十六	八八六十四
一九得九	二九十八	三九二十七	四九三十六	五九四十五	六九五十四	七九六十三	八九七十二	九九八十一

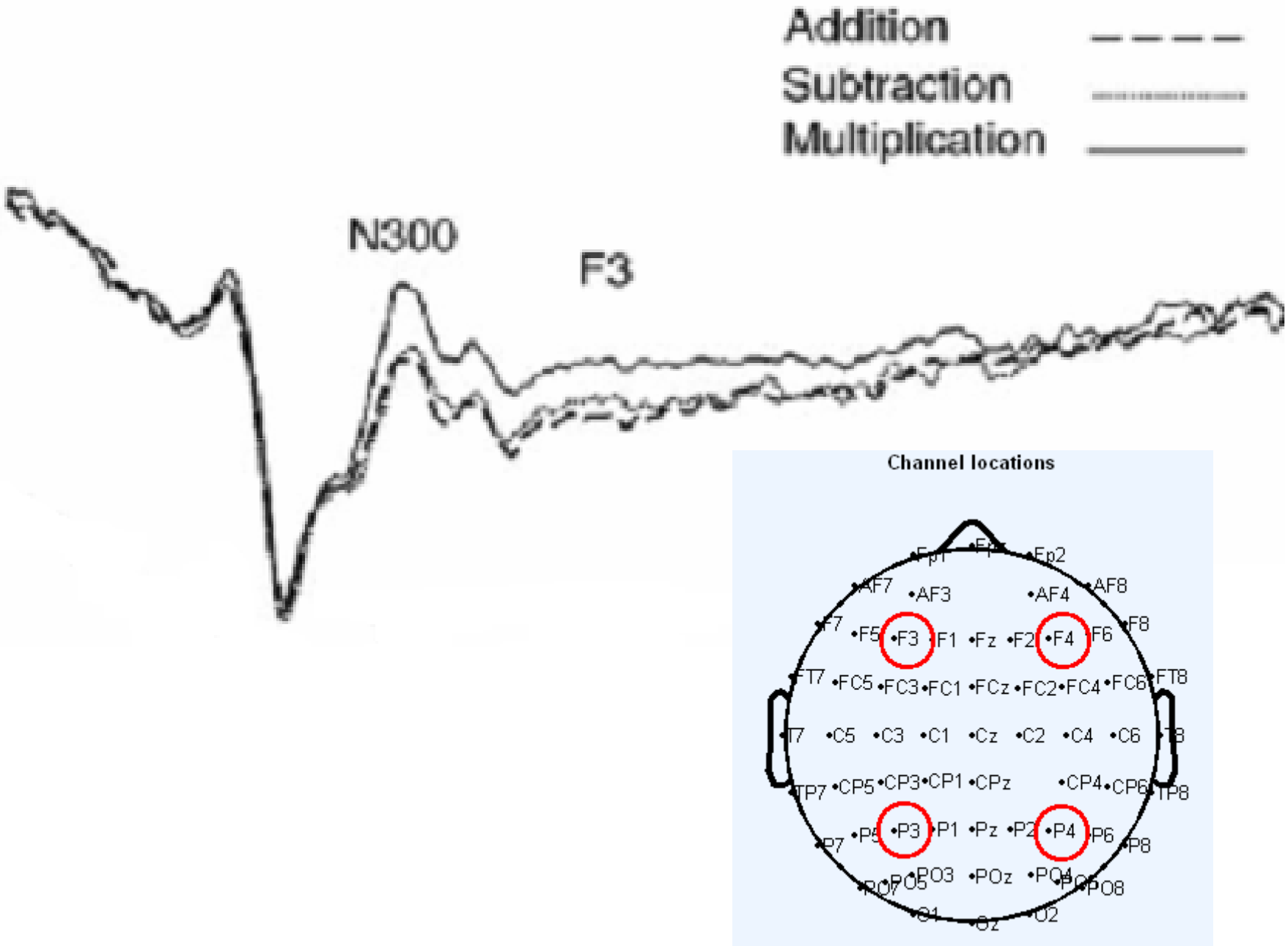
‘Small 9-9 table’ used in Mainland nowadays

- An ERP study to show the dissociation
  - Subjects: 18 Chinese undergraduates
  - Task (addition, subtraction and multiplication): Delayed arithmetic verification

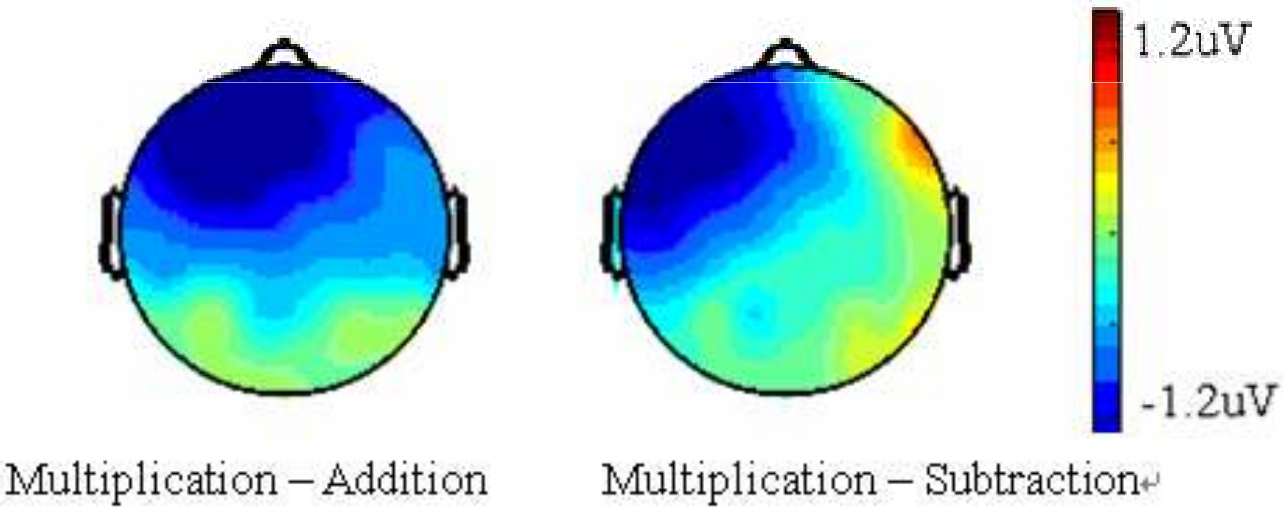
(Zhou et al., 2006, *Neuropsychologia*)


Addition
Subtraction
Multiplication
N300
F3
F4
P3
P4
-2.5uV
-200
0
100
500
1000
1500 ms


Channel locations
Fp2
AF7
AF3
AF4
AF8
F7
F5
F3
F1
Fz
F2
F4
F6
F8
FT7
FC5
FC3
FC1
FCz
FC2
FC4
FC6
FT8
T7
C5
C3
C1
Cz
C2
C4
C6
T8
TP7
CP5
CP3
CP1
CPz
CP4
CP6
TP8
P7
P5
P3
P1
Pz
P2
P4
P6
P8
PO3
POz
PO4
PO8
O1
Oz
O2


Addition
Subtraction
Multiplication
N300
F3
Channel locations
Fp1
Fp2
AF7
AF3
AF4
AF8
F7
F5
F3
F1
Fz
F2
F4
F6
F8
FT7
FC5
FC3
FC1
FCz
FC2
FC4
FC6
FT8
T7
C5
C3
C1
Cz
C2
C4
C6
T8
TP7
CP5
CP3
CP1
CPz
CP4
CP6
TP8
P7
P5
P3
P1
Pz
P2
P4
P6
P8
PO7
PO5
PO3
POz
PO4
PO6
PO8
O1
Oz
O2

An ERP study to show the dissociation



- An fMRI study to show the dissociation
  - Subjects: 20 Chinese undergraduates
  - Task: Normal arithmetic verification
    - 4+3=8#, 8+3=11
    - 3x7=24, 6x9=54

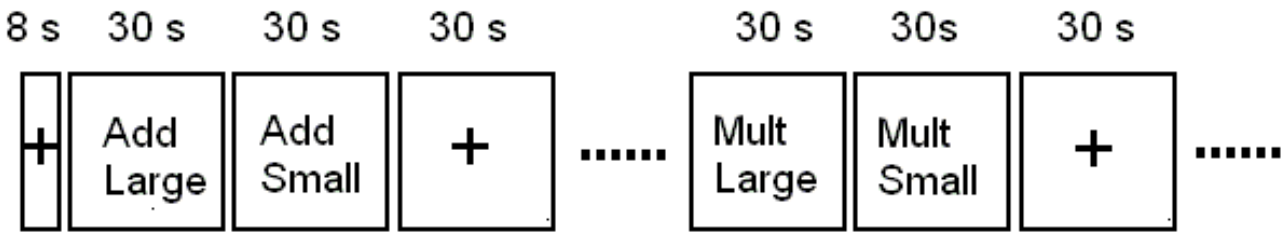
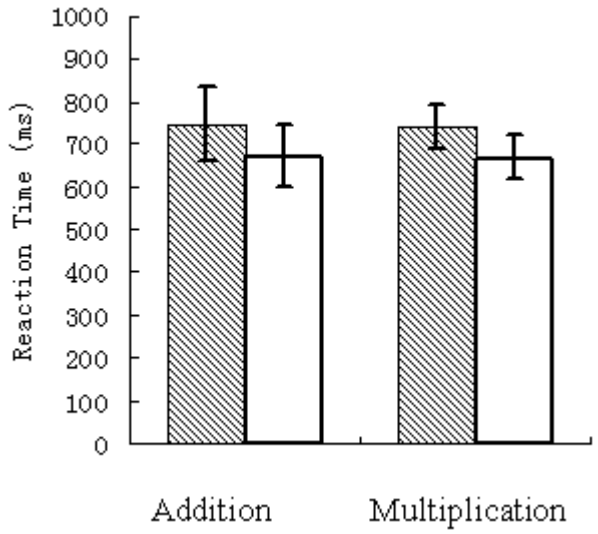
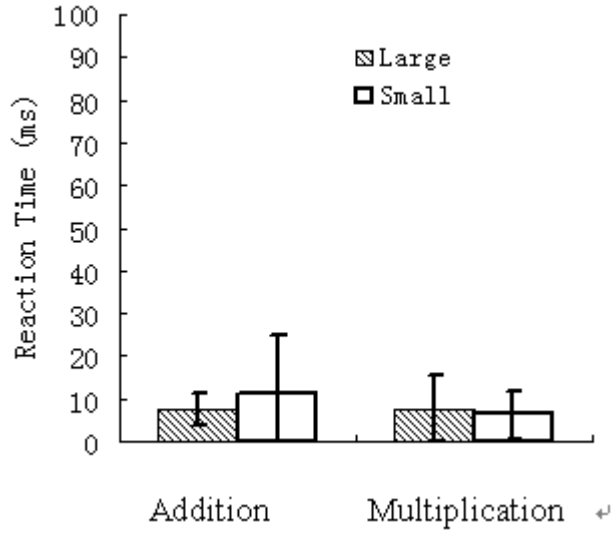


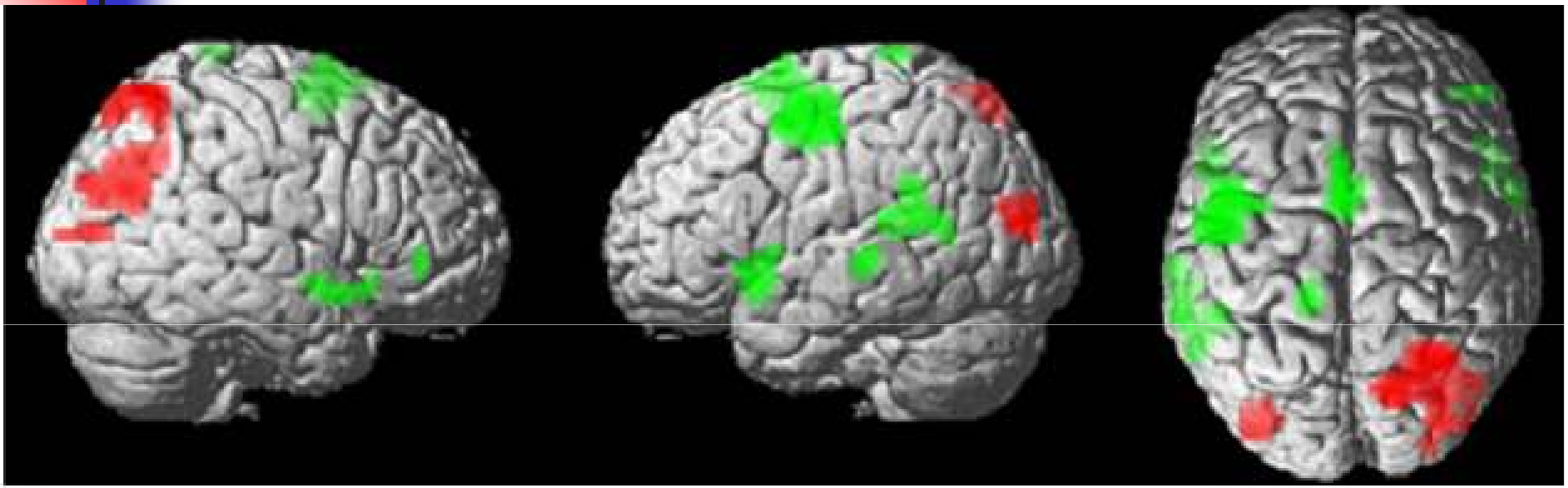
Figure: Schematic Representation for Experimental Design

(Zhou et al., 2007, *Neuroimage*)

- Results: Behavioral Data
  - There was problem size effect
  - No operation effect
  - No interaction effect







Green: more activation for multiplication
Red: More activation for addition

- Conclusion
  - The arithmetic brain could be shaped by learning experience during the acquisition of simple arithmetic.
  - There are the dissociated brain organizations in simple arithmetic: the brain regions for visuo-spatial processing and the brain regions for verbal processing.

The Operand-Order Effect in Single-Digit Multiplication: An ERP Study of Chinese Adults

- Operand order effect in multiplication
  - subjects took shorter time to respond to the smaller-operand-first problems (e.g., 3×7) than to the larger-operand-first problems (e.g., 7×3).

- Only undergraduates from Mainland other than from North America showed the operand order effect (LeFevre & Liu, 1997; Lefevre et al., 2001; Zhou & Dong, 2003).

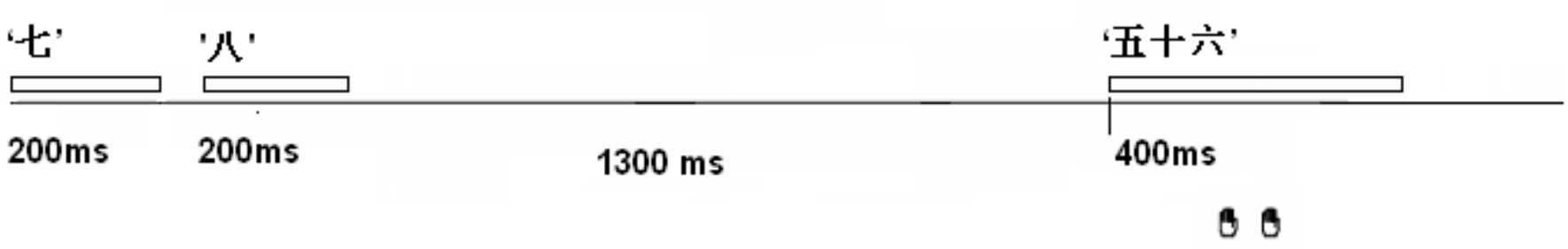
Multiplication table used in Mainland and Taiwan

一一得一							
一二得二二二得四							
一三得三二三得六	三三得九						
一四得四二四得八	三四十二	四四十六					
一五得五二五一十	三五十五	四五二十	五五二十五				
一六得六二六十二	三六十八	四六二十四	五六三十	六六三十六			
一七得七二七十四	三七二十一	四七二十八	五七三十五	六七四十二	七七四十九		
一八得八二八十六	三八二十四	四八三十二	五八四十	六八四十八	七八五十六	八八六十四	
一九得九二九十八	三九二十七	四九三十六	五九四十五	六九五十四	七九六十三	八九七十二	九九八十一

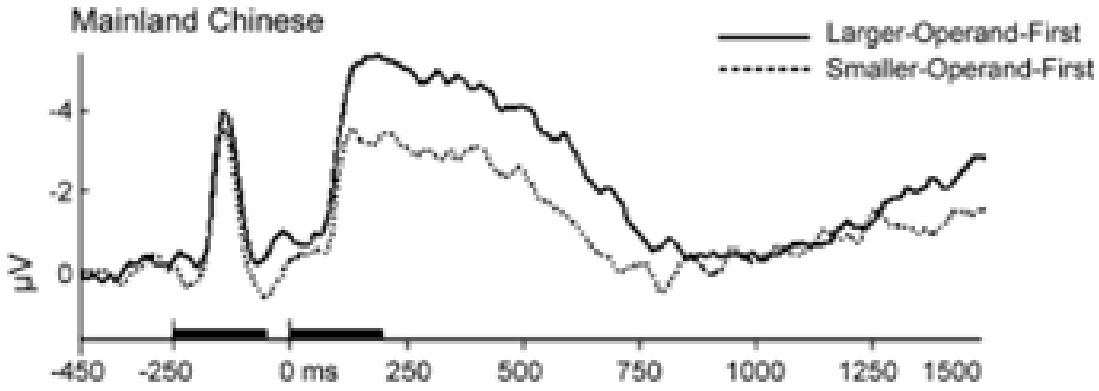
Multiplication table used in Hong Kong and Macau

一一得一二一得二	三一得三	四一得四	五一得五	六一得六	七一得七	八一得八	九一得九
一二得二二二得四	三二得六	四二得八	五二一十	六二十二	七二十四	八二十六	九二十八
一三得三二三得六	三三得九	四三十二	五三十五	六三十八	七三二十一	八三二十四	九三二十七
一四得四二四得八	三四十二	四四十六	五四二十	六四二十四	七四二十八	八四三十二	九四三十六
一五得五二五一十	三五十五	四五二十	五五二十五	六五三十	七五三十五	八五四十	九五四十五
一六得六二六十二	三六十八	四六二十四	五六三十	六六三十六	七六四十二	八六四十八	九六五十四
一七得七二七十四	三七二十一	四七二十八	五七三十五	六七四十二	七七四十九	八七五十六	九七六十三
一八得八二八十六	三八二十四	四八三十二	五八四十	六八四十八	七八五十六	八八六十四	九八七十二
一九得九二九十八	三九二十七	四九三十六	五九四十五	六九五十四	七九六十三	八九七十二	九九八十一

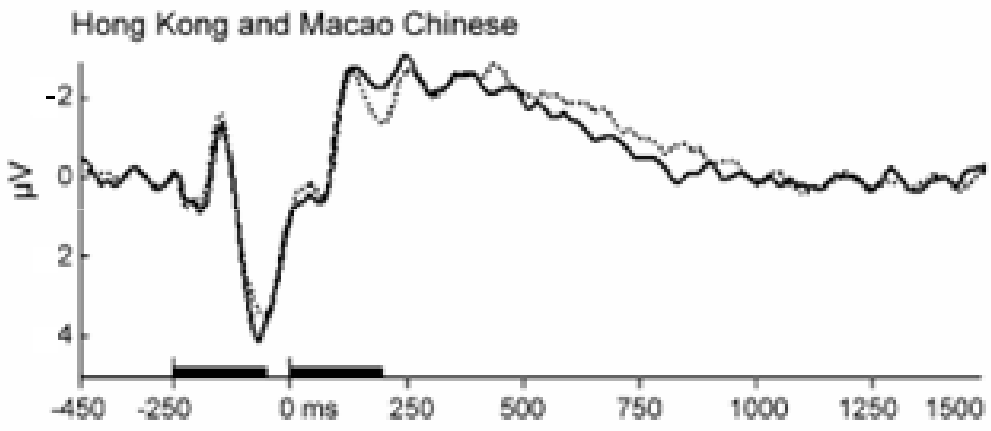
- Subjects
  - 20 mainland Chinese
  - 20 Hong Kong and Macao Chinese
- Procedure
  - The pronunciations of operands from 2 through 9 were presented in Mandarin for the Mainland Chinese subjects and in Cantonese (their native dialect) for Hong Kong and Macao subjects.
  - Each operand was presented for 200 ms. For each trial, the first operand was first presented, followed by a 50 ms silence and then by the second operand.

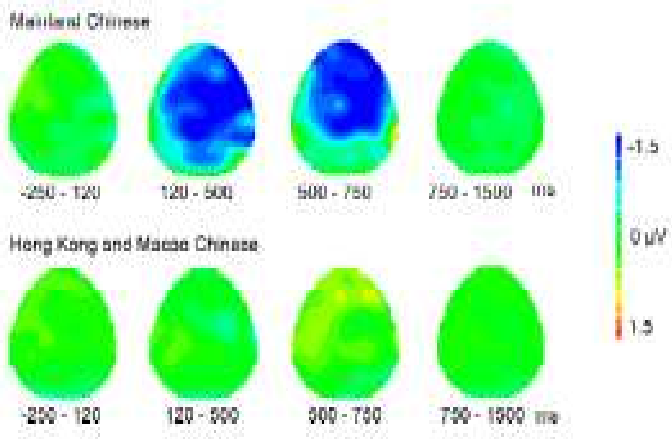


Paradigm used in the study (7x8=56 as example)



(Zhou et al., 2007)





Only Mainland Chinese showed the operand-order effect

(Zhou et al., 2007, *Neuroscience letters*)

- The larger-operand-first problems elicited greater negative potentials across representative electrodes of the whole scalp, emerging at about 120 ms after the onset of the second operand.
- These results suggest that the particular experience of acquiring multiplication facts had pronounced impact on their representations in the brain.

Thank you very much!